nature REVIEWS NEUROSCIENCE

SUPPLEMENTARY INFORMATION

In format as provided by the authors

Portraits of communication in neuronal networks

Gerald Hahn, Adrian Ponce-Alvarez, Gustavo Deco, Ad Aertsen and Arvind Kumar

https://doi.org/10.1038/s41583-018-0094-0

Portraits of communication in neuronal networks: Supplementary information

Gerald Hahn[1,*], Adrian Ponce-Alvarez[1], Gustavo Deco[1,2], Ad Aertsen[3,4], and Arvind Kumar[4,5*]

[1]Center for Brain and Cognition, Computational Neuroscience Group, Department of Information and Communication Technologies, Universitat Pompeu Fabra, 08018 Barcelona, Spain
[2]Institució Catalana de la Recerca i Estudis Avancats, Universitat Pompeu Fabra, 08010 Barcelona, Spain
[3]Faculty of Biology, University of Freiburg, 79104 Freiburg, Germany
[4]Bernstein Center Freiburg, University of Freiburg, 79104 Freiburg, Germany
[5]Computational Science and Technology, School of Electrical Engineering and Computer Science, KTH Royal Institute of Technology, Stockholm, 11428, Sweden
[*] Arvind Kumar: E-mail: arvkumar@kth.se, Gerald Hahn: E-mail: gerald.hahn@upf.edu

Oscillations and synchrony in neuronal network activity dynamics

Balance of excitation and inhibition shapes both population synchronization and oscillations – two key features of the global dynamics of spiking neuronal networks that affect the propagation of spiking activity within and between such networks.

Population oscillations

In a network of excitatory and inhibitory neurons, persistent excitation of the excitatory population and/or inhibition of the inhibitory population generates oscillations, whereas persistent inhibition of the excitatory population and/or excitation of the inhibitory population suppresses oscillations. This general rule can be derived from the dynamics of a firing rate-based model. A standard mean-field model with additive interactions (Wilson-Cowan model)[1] is described by:

$$\begin{aligned} \tau_E \frac{d}{dt} r_E &= -r_E + w_{EE} f_E(r_E) + w_{EI} f_I(r_I) + I_E + \eta_E \\ \tau_I \frac{d}{dt} r_I &= -r_I + w_{IE} f_E(r_E) + w_{II} f_I(r_I) + I_I \eta_I \end{aligned} \tag{1}$$

where r_a is the average firing rate and τ_a is the effective time constant of the a population; w_{ab} is the effective coupling strength from population b to a; f_a is the transfer function of the a population activity (e.g. sigmoid function); I_a and η_a are the external input and noise to the population a. The synaptic interaction matrix of the network is:

$$A = \begin{bmatrix} w_{EE} - 1 & w_{EI} \\ w_{IE} & w_{II} - 1 \end{bmatrix} \tag{2}$$

In the linear regime, eigenvalues of the matrix $\mathbf{A}$ determine the dynamics and stability of the network activity. For stability, the real part of the eigenvalues should be negative. When synaptic weights are altered in such a way that the eigenvalues of $\mathbf{A}$ become complex, the network makes the Andronov-Hopf bifurcation[2] and exhibits oscillations[1,3]. Typically, in a linear network of excitatory and inhibitory populations, the Andronov-Hopf bifurcation occurs when the effective excitation in the excitatory population exceeds the effective inhibition in the inhibitory population[4,5]. Rate-based models provide only a qualitative description and cannot be used to understand how neuronal and synaptic properties affect the oscillations, because there are no physiological correlates of variables such as τ_E and τ_I.

The estimation of the linear response of spiking neuronal networks is a better strategy to reveal how neuron and synapse properties affect the oscillation dynamics[4,6]. In this approach, the network response to a perturbation is studied. If the response to a perturbation is smaller than the perturbation itself, the network state is considered stable, because the network will eventually return to its original state. To test whether a network with a certain set of parameters will exhibit oscillations, the stability of the network activity is checked using an oscillatory perturbation at different frequencies (λ). Both the neuron membrane and the synapses act as linear filters, introducing a phase shift and amplitude attenuation in the response. For a stable oscillatory activity we require that the response is identical to the input perturbation in amplitude, frequency and phase. This condition can be written as[4]:

$$1 = A_{EE}(\lambda)(1 + A_{II}(\lambda)) - A_{II}(\lambda) - A_{EI}(\lambda) A_{IE}(\lambda) \tag{3}$$

where $A_{ab}(\lambda) = w_{ab}R_a(\lambda)S_{ab}(\lambda)$, w_{ab} is the magnitude of the post-synaptic potential from population b to a, the frequency-domain transfer functions of the neurons are $R_a(\lambda)$ and of the synapses are $S_{ab}(\lambda)$; λ is the frequency of the oscillatory input perturbation. $R_a(\lambda)$ and $S_{ab}(\lambda)$ capture the fact that both neurons and synapses act as low-pass filters.

When the above condition (Eq. 3) holds, oscillations arise. Because the network operates in an inhibition-dominated regime, the firing rates of individual neurons can be smaller than the oscillation frequency and a different set of neurons participates in each oscillation cycle. In this sense, these oscillations are better characterized as stochastic oscillations (SO)[4,6]. The oscillation frequency of such oscillations is primarily determined by both synaptic delays and membrane time constant, but can be modulated by changing the relative strength of inputs.

Eq. 3 implies that all four types of interactions between the excitatory and inhibitory neurons ($E \to E$, $E \to I$, $I \to E$, $I \to I$) influence the network oscillations and a possible mismatch between excitation/inhibition amplitude and timing is the key to the emergence of oscillations. Only under specific conditions, oscillations may be mainly determined by inhibitory interactions alone (interneuron gamma:ING) or both excitatory and inhibitory interactions (pyramidal-interneuron gamma:PING)[7].

The linear response of spiking neuronal networks also reveals that, independently of the synaptic and neuronal parameters, the network activity regimes are determined by the effective excitatory drive and the ratio of recurrent inhibition and excitation (I/E ratio, Fig. 1Left)[4,6]. When inhibition and excitation are balanced, the network exhibits a non-oscillatory asynchronous-irregular regime (AI). In that case, spiking among neurons is asynchronous, because correlations due to shared inputs are actively canceled. Spike patterns are irregular, because spikes are driven by fluctuations in the synaptic input. As the I/E ratio is increased, the Andronov-Hopf bifurcation occurs and network oscillations emerge (SO state). In addition, synchrony may also emerge when correlations are not canceled. Thus, this activity regime is termed synchronous-irregular (SI, see below). (Fig. 1Left). As the effective excitation is increased, the bifurcation occurs at smaller values of the I/E ratio (or smaller recurrent inhibition)(Fig. 1Left).

Under healthy physiological conditions, the network operating point is close to the Andronov-Hopf bifurcation. That is, the network does not exhibit any oscillation, but an external input can induce damped oscillations with the following set of events: transient excitation of excitatory neurons increases recurrent inhibition, which suppresses the activity of both excitatory and inhibitory neurons. As a consequence of the reduced inhibitory activity, excitatory neurons are released from inhibition and spike at a higher activity. As this sequence of events repeats, oscillations emerge. All these events can be clearly observed at the level of individual neuron membrane potentials and synaptic conductances[8]. Because the network operates below the Andronov-Hopf bifurcation, the response magnitude progressively decreases, resulting in damped oscillations and network resonance (Fig. 1)[8,9]. The same set of events can be initiated by transiently inhibiting the inhibitory population[5].

As the operating point of the network moves closer to the bifurcation, the network response may also display more complex dynamics alongside oscillations[10]. Finally, we note that available analytical methods only provide insights into single frequency oscillations.

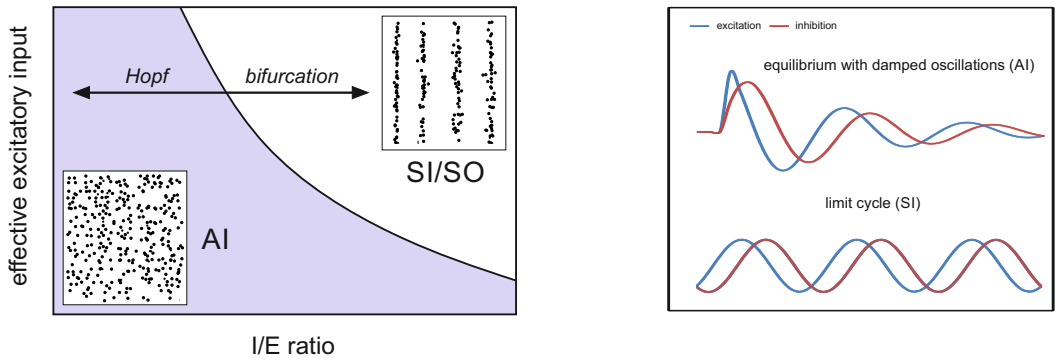

Figure 1. (**Left**) Schematic description of network activity regimes as a function of effective excitatory drive (y-axis) and the ratio of recurrent inhibition and excitation (x-axis). The AI and SI/SO regimes are observed in the shaded and non-shaded regions, respectively. Insets: examples of spiking activity raster displays in AI and SI/SO regimes, respectively. These rasters schematicaly show of AI and SI/SO activities. For actual spiking activity in AI and SI/SO state please refer to Kumar et al.[11]). (**Right**) Lower traces show a scheme of excitatory (blue) and inhibitory (red) population activity in the SI/SO state. The top traces show damped oscillations in the excitatory (blue) and inhibitory (red) population activity, when a network operating close to the Andronov-Hopf bifurcation is perturbed with a PP.

Pairwise correlations

The origin of synchrony lies in shared inputs. Such shared inputs may arise due to sharing of direct or indirect projections[12]. Even in sparsely connected random networks (with connection probability = 0.1) there are sufficient shared inputs to induce a population-wide synchronization[11]. However, recurrent inhibition (I/E ratio) can be tuned to cancel the bulk of pair-wise correlations and maintain an AI state. Consider a pair of excitatory neurons receiving shared input from excitatory pre-synaptic neurons. Because of the shared inputs, this pair of neurons will spike in a correlated fashion. However, the shared inputs can be canceled if the same shared input also arrives at the two neurons via shared inhibitory projections[13,14]. That is, when neurons share both excitatory and inhibitory inputs, the effect of shared input is suppressed, resulting in an asynchronous state. An imbalance between the degree of shared excitatory and inhibitory neurons (either due to unequal numbers, synaptic weights and/or firing rates) tends to synchronize the neuronal activity.

Both synchrony and oscillations can coexist in a neuronal network, because they arise due to a mismatch between excitation and inhibition (amplitude and/or time), but synchrony does not automatically imply oscillations and vice versa.

Entrainment and resonance in neuronal oscillations

Given the matrix $\mathbb{A}$, the network described by eq. 1 has an intrinsic oscillation frequency ($\omega_0 = 2\pi f_0$). When the system is driven by an oscillatory external input (e.g. $I_E = A_F sin(\omega_F t + \varphi_F) = A_F sin[\phi_F(t)]$), the network responds with an oscillation with amplitude R_{net} and phase $\phi_{net}(t)$. As the amplitude and frequency of the periodic input is systematically increased, two different but related phenomena can be observed in the steady state network response:

Entrainment: When the receiver network is operating in an oscillatory state, periodic input from another network can 'entrain' the oscillations of the receiver network[2]. It is necessary for entrainment that the input frequency (ω_F) is close to the network oscillations frequency (ω_0). When $\omega_0 \approx \omega_F$ the network gets entrained and the phase difference between input and network output remains constant over time ($\Delta\phi(t) = \phi_{net}(t) - \phi_F(t) = const.$). Entrainment can be observed only for a small range of $\Delta\omega$ ($= \omega_F - \omega_0$). This range of $\Delta\omega$ in which entrainment can occur increases as input amplitude (A_F) is increased. In the space spanned by $\Delta\omega$ and A_F, the region in which entrainment occurs is called the 'Arnold Tongue'[2] (Fig. 2a). Within the Arnold Tongue, the phase difference $\Delta\phi(t)$ converges to a fixed value after a few oscillation cycles (Fig. 2c). Outside the Arnold Tongue regime, $\Delta\phi(t)$ continues to change with time independently of the external input phase[15].

Resonance: When a network is tuned to operate close the bifurcation between oscillatory (SI/SO) and non-oscillatory (AI) states, periodic input can induce oscillations provided the input frequency matches (ω_F) with the oscillation frequency (ω_0) of the receiving network (Fig. 2d). This phenomenon is called resonance[2]. For a fixed A_F, the amplitude of the induced oscillatons (R_{net}) changes in a non-monotonic fashion as a function of $\Delta\omega$ (Fig. 2b).

Both entrainment and resonance require several cycles before the response magnitude, frequency and phase reach a fixed value. Finally, while entrainment and resonance often appear concomitantly, they are different phenomena. Entrainment is observed when the network is exhibiting persistent oscillations, whereas resonance can occur even when the network is not exhibiting any persistent oscillations.

Role of neuron types in shaping the oscillations and synchrony in a neuronal network

The neurons, in particular the inhibitory interneurons, in the brain are highly diverse in terms of their gene expression, chemical make-up, morphology, electrophysiology and connectivity[16–18]. Naturally the question arises how this neuronal diversity affects the transmission of spiking activity. Because neuronal networks in the brain are composed of different types of neurons, individual neuron properties can affect the transmission primarily by altering the network state.

The neuron transfer function features in Eq. 3 (the variable R_a). Hence, individual neuron types can affect the emergence of global oscillations, resonance and entrainment. Certain neuron types, e.g. those that spike in bursts, may also affect the network dynamics by altering the effective synaptic weight (because of the temporal summation of PSPs induced by a burst)[19]. Overall, the effect of different neuron types can be summarised in the excitation-inhibition balance (both amplitude and timing) which eventually determines the global activity state and transmission properties of a neuronal communication system. Finally, our previous work has shown that the impact of a neuron type on the global activity dynamics of a recurrent random network depends on the network activity regime itself[19].

However, how neuronal diversity affects the emergence of correlations and synchrony is not well understood. Existing computational and theoretical work has confirmed that the transfer of correlation (from input to output) depends on the spiking dynamics of the neurons and on the slope of the input-output transfer function of the neurons[20,21]. Neurons operating in a coincidence detection regime are more suited to transfer correlations than neurons operating in the 'integrator-mode'[21].

References

1. Wilson, H. R. & Cowan, J. D. Excitatory and inhibitory interactions in localized populations of model neurons. *Biophys. J.* **12**, 1–24 (1972). DOI 10.1016/S0006-3495(72)86068-5.

2. Pikovsky, A., Rosenblum, M., Kurths, J. & Kurths, J. *Synchronization: a universal concept in nonlinear sciences* (Cambridge university press, 2003).

3. Borisyuk, R. M. & Kirillov, A. B. Bifurcation analysis of a neural network model. *Biol. Cybern.* **66**, 319–325 (1992).

4. Ledoux, E. & Brunel, N. Dynamics of networks of excitatory and inhibitory neurons in response to time-dependent inputs. *Front. Comput. Neurosci.* **5**, 1–17 (2011). DOI 10.3389/fncom.2011.00025.

5. Kumar, A., Cardanobile, S., Rotter, S. & Aertsen, A. The role of inhibition in generating and controlling parkinson's disease oscillations in the basal ganglia. *Front. Syst. Neurosci.* **5**, 86 (2011). DOI 10.3389/fnsys.2011.00086.

6. Brunel, N. Dynamics of sparsely connected networks of excitatory and inhibitory spiking neurons. *J Comput. Neurosci* **8**, 183–208 (2000).

7. Tiesinga, P. & Sejnowski, T. J. Cortical enlightenment: Are attentional gamma oscillations driven by ing or ping? *Neuron* **63**, 727–732 (2009). DOI 10.1016/j.neuron.2009.09.009.

8. Hahn, G., Bujan, A. F., Frégnac, Y., Aertsen, A. & Kumar, A. Communication through resonance in spiking neuronal networks. *Plos Comp. Bio.* **10**, e1003811 (2014). DOI doi:10.1371/journal.pcbi.1003811.

9. Kang, K., Shelley, M., Henrie, J. A. & Shapley, R. Lfp spectral peaks in v1 cortex: network resonance and cortico-cortical feedback. *J Comput. Neurosci* **29**, 495–507 (2010). DOI 10.1007/s10827-009-0190-2.

10. Deco, G. & Kringelbach, M. L. Hierarchy of information processing in the brain: A novel 'intrinsic ignition' framework. *Neuron* **94**, 961–968 (2017). DOI 10.1016/j.neuron.2017.03.028.

11. Kumar, A., Schrader, S., Aertsen, A. & Rotter, S. The high-conductance state of cortical networks. *Neural Comput.* **20**, 1–43 (2008).

12. Pernice, V., Staude, B., Cardanobile, S. & Rotter, S. How structure determines correlations in neuronal networks. *PLoS Comp Biol* **7**, e1002059 (2011). DOI 10.1371/journal.pcbi.1002059.t001.

13. Renart, A. *et al.* The asynchronous state in cortical circuits. *Sci.* **327**, 587–90 (2010). DOI 10.1126/science.1179850.

14. Tetzlaff, T., Helias, M., Einevoll, G. T. & Diesmann, M. Decorrelation of neural-network activity by inhibitory feedback. *PLoS Comp Biol* **8**, e1002596 (2012). DOI 10.1371/journal.pcbi.1002596.t002.

15. Zheng, Z., Hu, G. & Hu, B. Phase slips and phase synchronization of coupled oscillators. *Phys. Rev. Lett.* **81**, 5318 (1998).

16. Klausberger, T. & Somogyi, P. Neuronal diversity and temporal dynamics: the unity of hippocampal circuit operations. *Sci.* **321**, 53–57 (2008).

17. DeFelipe, J. *et al.* New insights into the classification and nomenclature of cortical gabaergic interneurons. *Nat. Rev. Neurosci.* **14**, 202 (2013).

18. Jiang, X. *et al.* Principles of connectivity among morphologically defined cell types in adult neocortex. *Sci.* **350**, aac9462 (2015).

19. Sahasranamam, A., Vlachos, I., Aertsen, A. & Kumar, A. Dynamical state of the network determines the efficacy of single neuron properties in shaping the network activity. *Sci. reports* **6**, 26029 (2016).

20. De La Rocha, J., Doiron, B., Shea-Brown, E., Josić, K. & Reyes, A. Correlation between neural spike trains increases with firing rate. *Nat.* **448**, 802 (2007).

21. Ratté, S., Hong, S., De Schutter, E. & Prescott, S. A. Impact of neuronal properties on network coding: roles of spike initiation dynamics and robust synchrony transfer. *Neuron* **78**, 758–772 (2013).

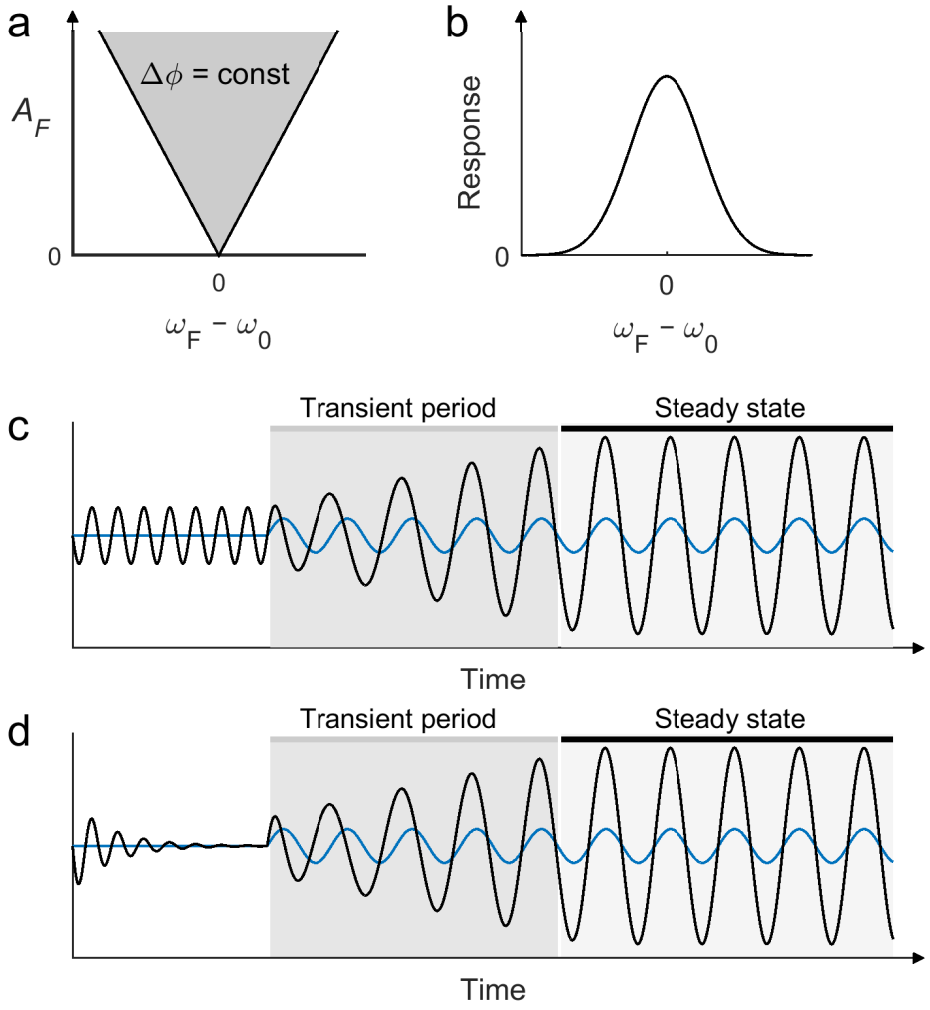

Figure 2. Schematic description of network entrainment and resonance. (**a**) The shaded region (Arnold Tongue) marks the range of A_F and $\Delta\omega$ for which entrainment occurs. (**b**) Resonance curve: network response R_{net} as a function of $\Delta\omega$, with A_F fixed. (**c**) In the case when the network is tuned to operate in in an oscillatory state, if the external input has a frequency within the Arnold Tongue, within a few cycles network activity phase matches with that of the input (steady state of the system) and the network response magnitude reaches its maximal value. (**d**) A network operating close to the bifurcation between AI and SI/SO does not exhibit oscillations in its steady state but input perturbations can create a damped oscillation. In such a state when the input oscillation frequency is close to the intrinsic frequency of the network, the input induces oscillation in the receiver network. In other words, the network resonates when the input frequency matches its intrinsic oscillation frequency. It is important to note that entrainment and resonance stabilize after some transient time as shown in panels **c** and **d**.